88107402

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Friday 5 November 2010 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- A graphic display calculator is required for this paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 24]*

Part A

100 students are asked what they had for breakfast on a particular morning. There were three choices: cereal (X), bread (Y) and fruit (Z). It is found that

10 students had all three
17 students had bread and fruit only
15 students had cereal and fruit only
12 students had cereal and bread only
13 students had only bread
8 students had only cereal
9 students had only fruit

(a) Represent this information on a Venn diagram. *[4 marks]*

(b) Find the number of students who had none of the three choices for breakfast. *[2 marks]*

(c) Write down the percentage of students who had fruit for breakfast. *[2 marks]*

(d) Describe in words what the students in the set $\boldsymbol{X} \cap \boldsymbol{Y}'$ had for breakfast. *[2 marks]*

(e) Find the probability that a student had **at least** two of the three choices for breakfast. *[2 marks]*

(f) Two students are chosen at random. Find the probability that both students had all three choices for breakfast. *[3 marks]*

(This question continues on the following page)

(Question 1 continued)

Part B

The same 100 students are also asked how many meals on average they have per day. The data collected is organized in the following table.

	3 or fewer meals per day	**4 or 5 meals per day**	**More than 5 meals per day**	**Total**
Male	15	25	15	55
Female	12	20	13	45
Total	27	45	28	100

A χ^2 test is carried out at the 5 % level of significance.

(a) Write down the null hypothesis, H_0, for this test. *[1 mark]*

(b) Write down the number of degrees of freedom for this test. *[1 mark]*

(c) Write down the critical value for this test. *[1 mark]*

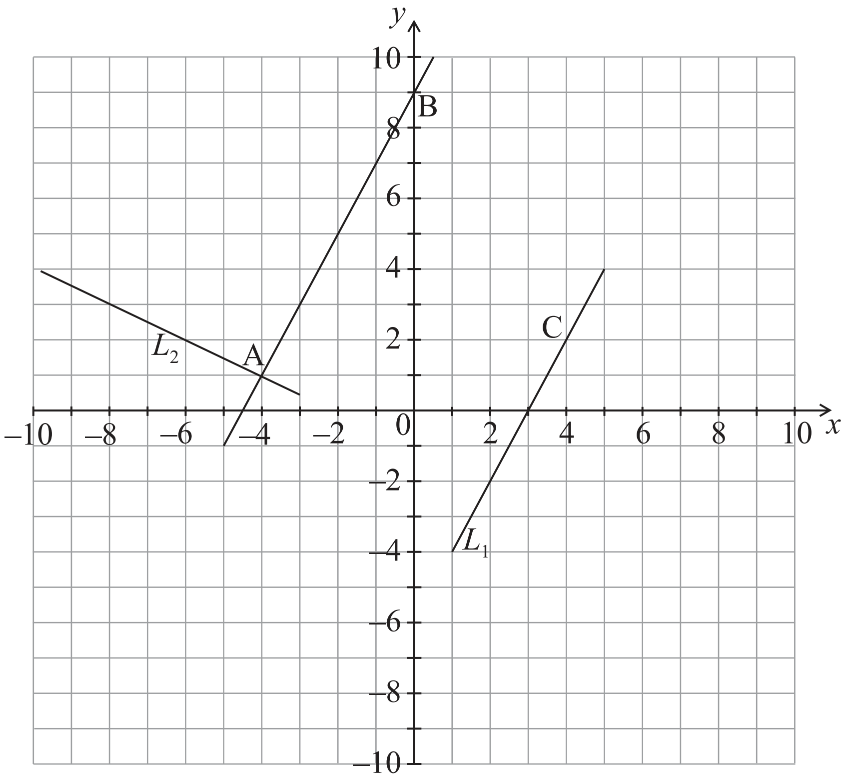
(d) Show that the expected number of females that have more than 5 meals per day is 13, correct to the nearest integer. *[2 marks]*

(e) Use your graphic display calculator to find the χ^2_{calc} for this data. *[2 marks]*

(f) Decide whether H_0 must be accepted. Justify your answer. *[2 marks]*

2. *[Maximum mark: 13]*

The points $\mathrm{A}(-4,1)$, $\mathrm{B}(0,9)$ and $\mathrm{C}(4,2)$ are plotted on the diagram below. The diagram also shows the lines AB, L_1 and L_2.



(a) Find the gradient of AB. *[2 marks]*

L_1 passes through C and is parallel to AB.

(b) Write down the y-intercept of L_1. *[1 mark]*

L_2 passes through A and is perpendicular to AB.

(c) Write down the equation of L_2. Give your answer in the form $\boldsymbol{ax}+\boldsymbol{by}+\boldsymbol{d}=0$ where a, b and $\boldsymbol{d} \in \mathbb{Z}$. *[3 marks]*

(d) Write down the coordinates of the point D, the intersection of L_1 and L_2. *[1 mark]*

There is a point R on L_1 such that ABRD is a rectangle.

(e) Write down the coordinates of R. *[2 marks]*

The distance between A and D is $\sqrt{45}$.

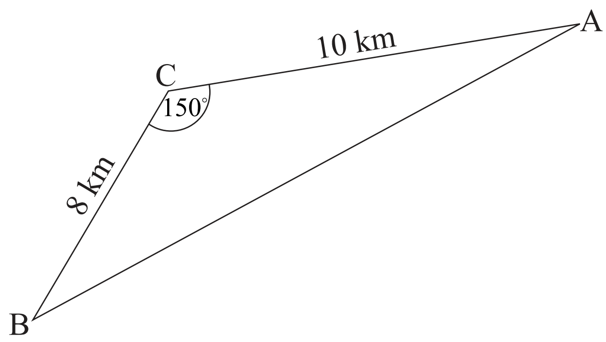
(f) (i) Find the distance between D and R.

(ii) Find the area of the triangle BDR. *[4 marks]*

3. *[Maximum mark: 16]*

In the diagram below A, B and C represent three villages and the line segments AB, BC and CA represent the roads joining them. The lengths of AC and CB are 10 km and 8 km respectively and the size of the angle between them is 150°.

diagram not to scale

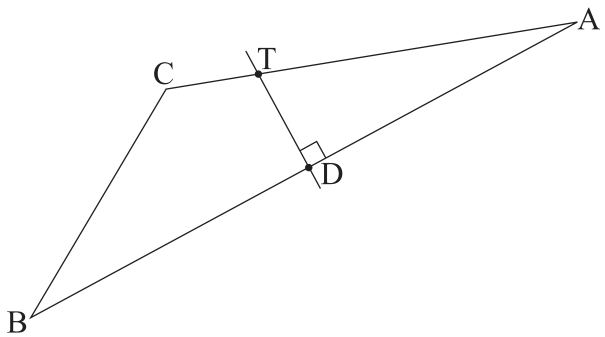


(a) Find the length of the road AB. *[3 marks]*

(b) Find the size of the angle CAB. *[3 marks]*

Village D is halfway between A and B. A new road perpendicular to AB and passing through D is built. Let T be the point where this road cuts AC. This information is shown in the diagram below.

diagram not to scale



(c) Write down the distance from A to D. *[1 mark]*

(d) Show that the distance from D to T is 2.06 km correct to three significant figures. *[2 marks]*

A bus starts and ends its journey at A taking the route AD to DT to TA.

(e) Find the total distance for this journey. *[3 marks]*

The average speed of the bus while it is moving on the road is 70 km h^{-1}.
The bus stops for 5 **minutes** at each of D and T.

(f) Estimate the time taken by the bus to complete its journey. Give your answer correct to the nearest minute. *[4 marks]*

4. *[Maximum mark: 16]*

Give all answers in this question correct to two decimal places.

Part A

Estela lives in Brazil and wishes to exchange 4000 BRL (Brazil reals) for GBP (British pounds). The exchange rate is $1.00\text{ BRL} = 0.3071\text{ GBP}$. The bank charges 3 % commission on the amount in BRL.

(a) Find, **in BRL**, the amount of money Estela has after commission. *[2 marks]*

(b) Find, in GBP, the amount of money Estela receives. *[2 marks]*

After her trip to the United Kingdom Estela has 400 GBP left. At the airport she changes this money back into BRL. The exchange rate is now $1.00\text{ BRL} = 0.3125\text{ GBP}$.

(c) Find, in BRL, the amount of money that Estela should receive. *[2 marks]*

Estela actually receives 1216.80 BRL after commission.

(d) Find, in BRL, the commission charged to Estela. *[1 mark]*

(e) The commission rate is t %. Find the value of t. *[2 marks]*

Part B

Daniel invests \$1000 in an account that offers a nominal annual interest rate of 3.5 % **compounded half yearly**.

(a) Show that after three years Daniel will have \$1109.70 in his account, correct to two decimal places. *[3 marks]*

(b) Write down the interest Daniel receives after three years. *[1 mark]*

Helen invests \$1000 in an account that offers annual **simple interest**.

(c) Find the annual simple interest rate that would give Helen \$1109.70 after three years. *[3 marks]*

5. *[Maximum mark: 21]*

Consider the function $f(x) = x^3 - 3x^2 - 24x + 30$.

(a) Write down $f(0)$. *[1 mark]*

(b) Find $f'(x)$. *[3 marks]*

(c) Find the gradient of the graph of $f(x)$ at the point where $x = 1$. *[2 marks]*

The graph of $f(x)$ has a local maximum point, M, and a local minimum point, N.

(d) (i) Use $f'(x)$ to find the x-coordinate of M and of N.

(ii) Hence or otherwise write down the coordinates of M and of N. *[5 marks]*

(e) Sketch the graph of $f(x)$ for $-5 \le x \le 7$ and $-60 \le y \le 60$. Mark clearly M and N on your graph. *[4 marks]*

Lines L_1 and L_2 are parallel, and they are tangents to the graph of $f(x)$ at points A and B respectively. L_1 has equation $y = 21x + 111$.

(f) (i) Find the x-coordinate of A and of B.

(ii) Find the y-coordinate of B. *[6 marks]*